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INFLUENCE OF THE SHAPE AND GEOMETRIC PARAMETERS OF CLASPS ON THE STRENGTH AND HOLDING FORCE OF REMOVABLE ORTHOPEDIC DENTURES

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We propose a procedure for the evaluation of the characteristics of clasps of removable partial dentures made of thermoplastic materials. The procedure includes the following stages: evaluation of the specific pressure of a clasp on the retaining tooth, determination of the maximum internal forces and stresses in the clasp, and finding its shape in the unloaded state. The results of our investigations enable us to choose the optimal sizes of the clasp in order to create the required holding force for the denture with guaranteeing its sufficient strength.

Keywords: removable partial dentures, shape of a clasp, thermoplastics, internal forces, stresses.

The recovery of the integrity of dentoalveolar system is frequently realized with the help of removable partial dentures [4]. Acrylic plastics are the main materials used for the production of lamellar removable dentures. However, due to their low elasticity, low impact strength, and low fatigue strength, the products made of acrylic plastics are often destroyed in the course of chewing and as a result of falling onto the floor [14]. This explains more and more extensive application of thermoplastics, such as nylon, as materials of dentures. Unlike amorphous acrylic plastics, nylon is a crystalline polymer characterized by a low solubility, it is not toxic and completely safe for patients [13]. Dental nylons have good casting properties, low casting shrinkage, and insignificant moisture absorption. They can be easily treated and well polished. The dentures made of nylon have high esthetic characteristics [13]. However, as compared to acrylic plastics, nylon has lower strength and stiffness [11, 12, 15]. Thus, the moduli of elasticity of acrylic plastics and nylon in bending vary within the limits 2.4–3.6 GPa and 0.87–1.7 GPa, respectively. The ultimate strengths of acrylic plastics and nylon are equal to 64–91 MPa and 36–55 MPa, respectively [12]. The lower stiffness of nylon and its sufficiently high strength properties make it possible to produce from nylon not only the base but also fixing elements of a denture, in particular, holding clasps. In order that the clasps of lamellar dentures be reliable, their sizes should be properly substantiated. In choosing the sizes and shapes of the clasps, it is necessary to apply the methods of mechanical-mathematical simulation that have recently found increasing applications in various areas of dental treatment and are capable of increasing its efficiency [1–5, 7, 10].

In Fig. 1, we show a lamellar denture formed by base ***1*** with artificial teeth *2*. Base ***1*** is supported by a part of the alveolar branch of the jaw in which the teeth are absent and held by clasps ***3***. The base and clasps are manufactured from thermoplastics as a single whole. In the course of fixation on a holding tooth, the clasp is elastically deformed and creates the force of fixation of the denture. The denture is fixed on the holding teeth by a friction force F acting between the denture and the tooth.

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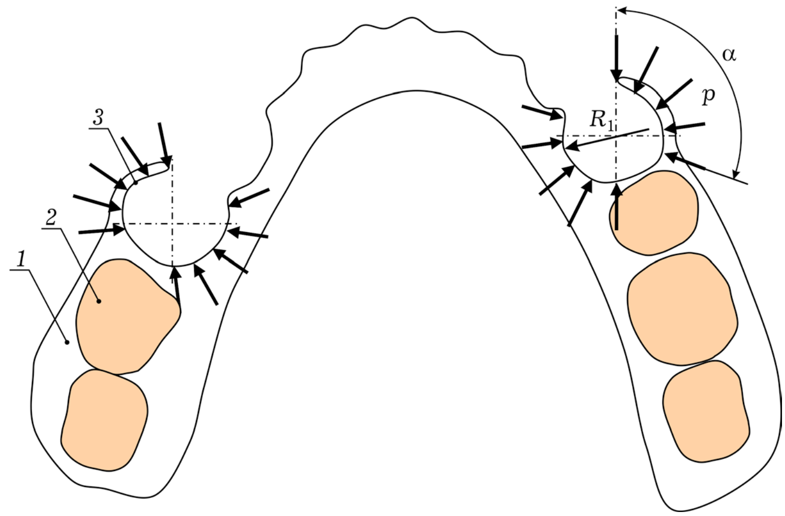

Fig. 1

The denture depicted in Fig. 1 is fixed by two clasps with the angle of contact with the holding tooth equal to α. If we assume that the specific pressure p of the clasp upon the holding tooth is uniformly distributed and that the tooth surface have the form of a cylinder with radius R_1, then the force of fixation F of the denture on supporting teeth is equal to

$$F = 4\alpha R_1 kp \text{ [N]}, \qquad (1)$$

where k is the coefficient of sliding friction of nylon on the holding tooth.

By using relation (1), we can find the required specific pressure p of the clasp on the holding tooth for a given fixation force F:

$$p = F/4\alpha R_1 k \text{ [N/mm]}. \qquad (2)$$

The aim of the present work is to study the influence of the shape and sizes of the clasps of dentures made of thermoplastics on the efficiency of orthopedic dental treatment. To solve this problem, it is necessary to determine the shape of a denture clasp guaranteeing the uniformity of its pressure on the holding tooth. The problem considered in the present work can be regarded as a basic direct problem of the mechanics of deformable solids in which the sizes of the body and the forces acting upon it are given, whereas the stresses and displacements of its points in the process of deformation should be determined [6]. If the displacements of the body in the process of deformation are known, then we can find its shape prior to deformation. Since the displacements of points and deformations of the clasp are regarded as small, in the numerical analysis of the stress-strain state, we can assume that the shape of the clasp does not change in the process of loading.

In Fig. 2, we present a computational scheme for the clasp in the position, where it suffers the action of a uniformly distributed specific pressure p on the side of the tooth [4]. The clasp is restrained in the base of the denture. Note that the transverse sizes of the tooth vary within the range 6–12 mm and the thickness of the clasp wall h exceeds 1 mm. Therefore, if the radius of the inner surface of the clasp $R_1 = 5$ mm, then the ratio of h to R_1 is greater than 0.2, and the clasp can be regarded as a curved beam with high curvature.

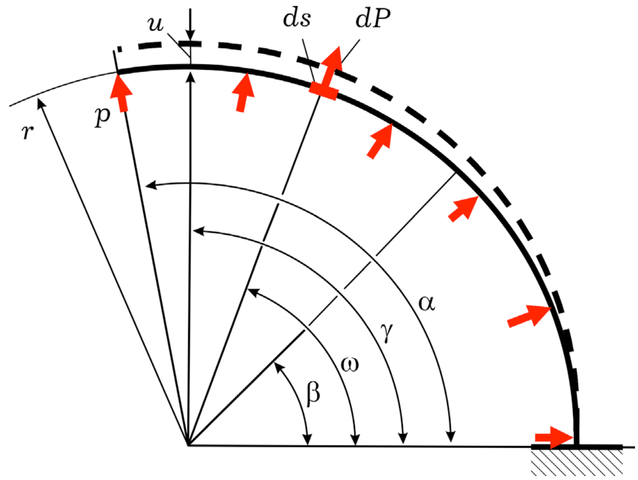

Fig. 2

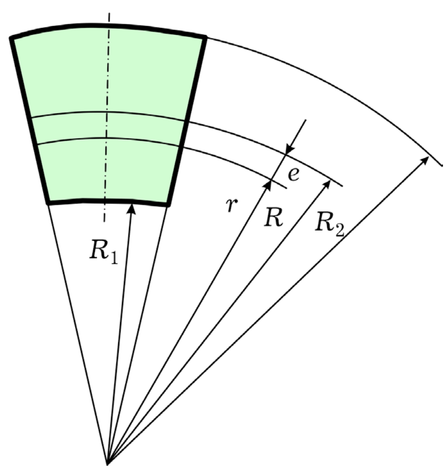

Fig. 3

The problem of determination of internal forces and displacements in a thin curved rod loaded by concentrated forces applied at the end was considered in [8]. Unlike [8, 9], we consider the problem of finding internal forces and displacements in a curved beam with high curvature under the action of uniformly distributed pressure.

We denote the radius of a neutral surface of the clasp, i.e., the surface whose material fibers are not deformed, by r. In order to determine the required thickness h and width b of the clasp, it is necessary to find the internal forces and stresses acting in the clasp body.

Under the action of a uniformly distributed load p, we observe the formations of a bending moment M, a tensile force N, and a shear force Q in the cross-sections of the curved beam. We determine these quantities in a section of the curved beam oriented at an arbitrary angle β. We consider the stress-strain state of the clasp in a polar coordinate system (ρ, ω). An elementary area of the curved beam of length $r\,d\omega$ oriented at an angle ω undergoes the action of an elementary force dP directed in the radial direction:

$$dP = pr\,d\omega. \tag{3}$$

We integrate the elementary forces dP within the limits from β to α. As a result, we get the internal forces M, N, and Q in a cross section oriented at an angle β:

$$M = \int_{\beta}^{\alpha} r \sin(\omega - \beta) pr\, d\omega = pr^2 [1 - \cos(\alpha - \beta)], \tag{4}$$

$$N = \int_{\beta}^{\alpha} \sin(\omega - \beta) pr\, d\omega = pr [1 - \cos(\alpha - \beta)], \tag{5}$$

$$Q = \int_{\beta}^{\alpha} \cos(\omega - \beta) pr\, d\omega = pr \sin(\alpha - \beta). \tag{6}$$

The bending moment and the longitudinal force attain their maximum values M_m and N_m in a section of the beam oriented at the angle $\beta = 0$. At the same time, the shear force Q_m is maximum in a section oriented at the angle $\beta = \alpha - \pi/2$:

$$M_m = pr^2(1 - \cos\alpha), \qquad N_m = pr(1 - \cos\alpha), \qquad Q_m = pr. \tag{7}$$

In Fig. 4, we present the plots of the internal moment and forces M, N, and Q acting in sections of the analyzed curved beam.

The internal moment M and force N induce normal stresses in the cross-section of the beam. The normal stresses caused by the action of N are uniformly distributed over the cross section of the curved beam. At the same time, the stresses caused by M are distributed according to the hyperbolic law. In curved beams, the neutral surface of their cross section does not pass through the center of gravity, which has the coordinate R. This is actually the surface with coordinate r located at the distance $e = R - r$ from the center of gravity (Fig. 3). If the cross section of the clasp has a rectangular shape bounded by the outer radius R_2, then, for the radius r, we obtain [8]

$$r = \frac{h}{\ln(R_2/R_1)}.$$

The normal stresses caused by the bending moment and formed on the inner and outer surfaces of the clasp are, respectively, tensile, σ_{mp}, and compressive, σ_{mc}, [8]:

$$\sigma_{\mathrm{mp}} = \frac{M_m}{eS} \frac{r - R_1}{R_1}, \qquad \sigma_{\mathrm{mc}} = \frac{M_m}{eS} \frac{r - R_2}{R_2}, \tag{8}$$

where $S = hb$ is the cross-sectional area of the clasp.

The normal stresses σ_p caused by the action of the longitudinal force N_m are as follows:

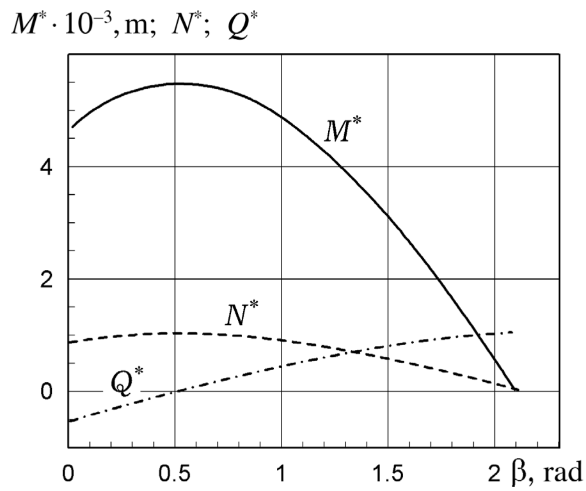

Fig. 4

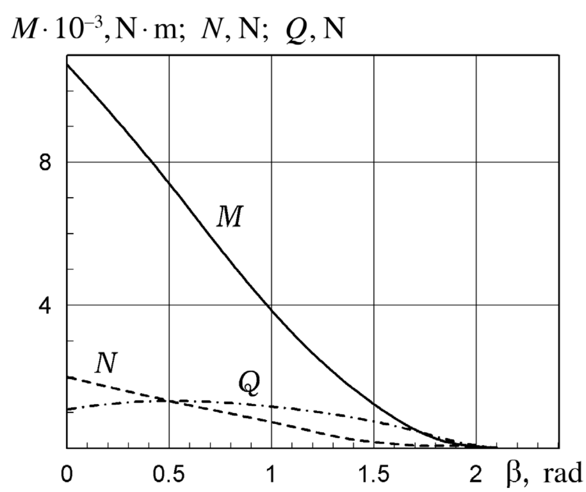

Fig. 5

$$\sigma_p = \frac{N_m}{S}. \tag{9}$$

The total normal stresses on the inner σ_1 and outer σ_2 surfaces of the clasp are, respectively,

$$\sigma_1 = \sigma_{\text{mp}} + \sigma_p \quad \text{and} \quad \sigma_2 = \sigma_{\text{mc}} + \sigma_p. \tag{10}$$

The action of the shear force Q_m induces tangential stresses. For the rectangular cross section of the clasp, they are given by the formula

$$\tau_m = \frac{1.5\,Q_m}{S}. \tag{11}$$

Since the tangential stresses are low as compared with normal stresses and reach the maximum value on the neutral axis of the section of a clasp, we do not take them into account in our strength analysis. The main hazard is connected with the action of total tensile stresses on the inner surface of the clasp.

The admissible stress $[\sigma]$ for nylon is given by the formula

$$\sigma_1 < [\sigma] = \frac{\sigma_B}{n} = 18 \text{ MPa}. \tag{12}$$

where $\sigma_B = 45$ MPa is the ultimate strength of nylon [11] and $n = 2.5$ is the safety margin [8].

In Table 1, we present the maximum admissible values of tensile stresses formed in the hazardous cross section of the clasp for the following denture-holding forces: $F = 5$ N, $F = 2$ N, and $F = 1$ N and various values of thickness h and width b of the clasp; $R_1 = 5$ mm, $k = 0.1$, and $\alpha = 2/3\,\pi$.

Table 1.

Holding force								
5 N			2 N			1 N		
b, mm	h, mm	σ_1, MPa	b, mm	h, mm	σ_1, MPa	b, mm	h, mm	σ_1, MPa
4.5	2.5	16.4	3.0	2.0	15.3	3.0	1.5	12.1
4.0	3.0	15.8	2.5	2.5	13.1	2.5	1.5	14.5
3.5	3.0	18.0	2.0	2.5	16.4	2.2	1.5	16.5

We now determine the shape of a clasp required to get a uniform distribution of its pressure upon the tooth p. It is assumed that, prior to application of a uniformly distributed load p, the inner surface of the clasp has the form of a circular cylinder of radius R_1. We now find the displacements of points of the clasp Δ_γ in a section oriented at an arbitrary angle γ under the action of a uniformly distributed load p by using the Maxwell–Mohr formula [8]:

$$\Delta_\gamma = \int_0^\gamma \frac{M(\beta)M^*(\beta)r}{EJ}\,d\beta + \int_0^\gamma \frac{N(\beta)N^*(\beta)r}{ES}\,d\beta + \int_0^\gamma \frac{Q(\beta)Q^*(\beta)r}{GS}\,d\beta, \tag{13}$$

where $M^*(\beta)$, $N^*(\beta)$, and $Q^*(\beta)$ are, respectively, the bending moment, the longitudinal force, and the shear forces caused by the unit load P applied to a point of the clasp at the angle γ in any section oriented at an angle β;

$$J = \frac{bh^3}{12} + bhe^2$$

is the moment of inertia of a section of the clasp; E is Young's modulus;

$$G = \frac{E}{2(1+\nu)}$$

is the shear modulus, and ν is Poisson's ratio of the material of the clasp.

The dependences of the bending moment $M^*(\beta)$ and the longitudinal $N^*(\beta)$ and shear $Q^*(\beta)$ forces on P take the form

$$M^*(\beta) = r \sin(\gamma - \beta), \tag{14}$$

$$N^*(\beta) = \sin(\gamma - \beta), \tag{15}$$

$$Q^*(\beta) = \cos(\gamma - \beta). \tag{16}$$

The dependences of $M^*(\beta)$, $N^*(\beta)$, and $Q^*(\beta)$ in the case where the unit load P is applied at an angle $\gamma = 2\pi/3$ are shown in Fig. 5.

Substituting relations (4)–(6) and (14)–(16) in (13) and integrating over the angle β, we get the following dependence of radial displacements of the clasp Δ_γ in a section oriented at the angle γ on the uniformly distributed load p:

$$\Delta_\chi = pr^2 \left[\frac{A}{E} \left(\frac{1}{S} + \frac{r^2}{J} \right) + \frac{B}{GS} \right], \tag{17}$$

where

$$A = 1 - \cos\gamma - 0.5\cos(\alpha+\gamma)\sin^2\gamma$$

$$+ 0.25\sin 2\gamma \sin(\alpha-\gamma) - 0.5\gamma\sin(\alpha+\gamma),$$

$$B = 0.5\gamma\sin(\alpha-\gamma) - 0.5\cos(\alpha+\gamma)\sin^2\gamma + 0.25\sin 2\gamma\sin(\alpha+\gamma).$$

In Table 2, we present the results of evaluation of the displacements Δ_γ of points of the clasp located at an angle γ under the action of a uniformly distributed load p. In our calculations, we took $\alpha = 2/3\,\pi$, $R_1 =$ 5 mm, $b = 2.2$ mm, $h = 1.5$ mm, $E = 1$ GPa, and $\nu = 0.35$. In Fig. 2, the deformed position of the clamp axis is shown by the dashed line.

Table 2.

Radial displacements Δ_γ of points of the clasp as functions of the angle γ

γ, degree	15	30	45	60	75	90	105	120
Δ_γ, mm	0	0.01	0.02	0.04	0.05	0.07	0.08	0.1

Thus, in order to determine the shape of the clasp of a removable lamellar denture made of thermoplastics, it is necessary to find, for given holding forces, the specific pressure of the clasp upon the holding tooth by using relation (2). After choosing proper cross-sectional sizes of the clasp, we determine the maximum tensile stresses in the hazardous cross section of the clasp by using relations (7)–(10). If the chosen section of the clasp satisfies the condition of strength (12), then we can find elastic displacements for all sections of the clasp according to relation (17). The sizes of the clasp in the unloaded state are determined as the difference between the radial size of the tooth and the displacement of the corresponding section of the clasp given by relation (17).

In recent years, removable dentures made of nylon are extensively used for the orthopedic denture treatment. The mechanical properties of nylon enable one to use it for the production not only of the base of denture but also of the clasps aimed at fixation of dentures on the holding teeth. The choice of a section of the clasp is performed according to the condition of its strength in the section, where the tensile stresses attain the maximum. The shape of a clasp must guarantee the uniform (over the surface of the tooth) distribution of its pressure upon the tooth. The numerical analysis of the clasp includes the determination of its displacements and the reconstruction of its shape in the unloaded state according to the data on displacements. The proposed procedure enables one to find the geometry of clasps satisfying the criteria of strength and reliability of fixation of dentures on the supporting teeth. The strength and reliability of lamellar dentures made of thermoplastic materials are important parameters of the efficiency of orthopedic treatment.

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