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SUPPORT REACTIONS OF REMOVABLE PARTIAL DENTURES*

V. L. Bogdanov[1], O. Ya. Grigorenko[1*], V. O. Malanchuk[2], G. V. Sorochenko[2***], and M. M. Tormakhov[1]**

The problem of preventing further degradation of a dental system with distally unlimited defects of dentition is considered. To prevent complications in orthopedic treatment when planning, it is necessary to estimate the value of the support reactions of a prosthesis caused by the chewing forces. In studying the problem, mechanical-mathematical modeling methods are used. To replace extended distally unlimited defects of dental rows, double-sided prostheses are in need. To study the stress state of a prosthesis–jaw system, a spatial model is used. The support reactions of extended distally unlimited removable partial prostheses with rigid and labile lock connections are determined. It is shown that the support reactions of such prostheses are distributed more uniformly then those with labile locks.

Keywords: spatial model, removable partial dentures, locking devices, support reactions, chewing load

Introduction. One of the most important dentistry problems is removing defects in dental rows by replacing missing teeth with artificial prostheses. The defects in dental rows are divided into limited ones between teeth and distally unlimited at the periphery of a dental row. Double-sided distally unlimited defects of dental rows, which are replaced by removable partial prostheses in orthopedic dentistry, are supposed to be the most complicated [3, 7–10, 12]. In this case, the reliability of prosthesis fixation, durability, and integrity of the dental system depends on the value of the support reactions of the prosthesis, that is, on the forces exerted by the prosthesis on the support teeth and the prosthesis bed. The values of these reactions depend also on the place the chewing force is applied to, the topography of dentition defects, the accuracy of producing and structure of the prosthesis as well as on the material the prosthesis is made of. The fixation of the prosthesis to a jaw may be realized using either klammers [1, 3] or rigid or labile locking devices [11, 12, 15]. If the defects of dental rows are small and limited, the functional overloading of the prosthesis supports is usually absent because the survived dental row readily compensates the lost function. When the defects spread and especially are distally unlimited, the overloading of the support elements increases considerably and may result either in the loss of the support teeth or resorption of alveolar process of the jaw. To avoid the subsequent degradation of the dental system with distally unlimited defects of dental rows even at the design stage, it is necessary to estimate the value of the possible reactions of the prosthesis supports during operation.

At present dental orthopedics widely uses methods of mathematical modeling [1, 2, 4–6, 13, 14, 16]. The plane stress–strain state of the prosthesis–jaw system, when the length of the defects of the dental row is small, is studied in [4, 5]. However, in the case of extended defects of the dental row, whose replacement makes it necessary to use double-sided prostheses, the stress–strain state of the prosthesis–jaw system considerably differs from the plane one. The present work

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[1] S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine, 03057; *e-mail: ayagrigorenko1991@gmail.com. [2] Bogomolets National Medical University, 34 Peremogy Av., Kyiv, Ukraine, 03057; e-mail: **malanchuk_v_a@ukr.net, ***anticaries15@gmail.com. Translated from Prikladnaya Mekhanika, Vol. 58, No. 2, pp. 31–38, March–April 2022. Original article submitted April 4, 2021.

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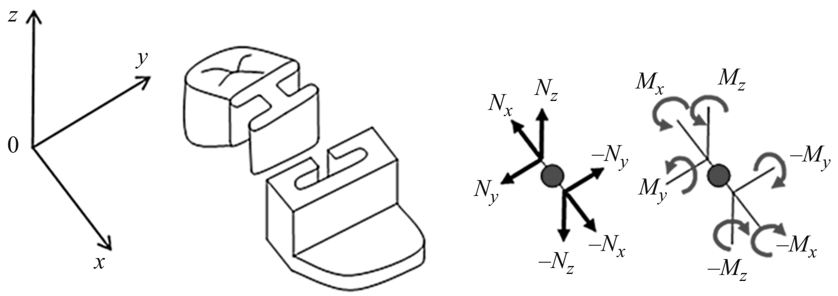


Fig. 1

addresses determining, using a spatial problem statement, the support reactions of extended unlimited distally double-sided removable partial prostheses with rigid and labile locking connections.

1. Problem Statement. In determining the support reactions of removable partial prostheses, we assume that the mechanical properties of all jaw tissues are described by Hooke's law, the prosthesis is made accurately, the prosthesis–jaw system is free of internal stresses, and the prosthesis with the chewing force being absent touches the support tooth and mucous tunic, but does not press on them.

The prosthesis–jaw system is statically indeterminate and the reactions of the prosthesis supports are determined with the displacement method. To this end, the rigidity of such components of the system as the prosthesis, tooth, mucous tunic placed under the prosthesis bed and periodontium tissues (periodontium is the layer of elastic tissue of 0.2–0.25 mm width connecting the tooth to the jaw bone) must be determined. By the rigidity is meant the ratio of the force to the displacement caused by it. Since the rigidity of the skeleton of the prosthesis and teeth is considerably higher than that of the other components of the prosthesis–jaw system [2], their deformations in comparison with those of mucous tunic and periodontium tissues [2, 6] can be neglected.

In accordance with the Viola designation system recommended by WHO, teeth in stomatology are designated by two indices *i* and *j*. Here *i* is the number of the segment in a dental row: number "1" and "2" refer to the teeth of the upper jaw arranged on the right and on the left, respectively, while number "3" and "4" to the teeth of the lower jaw arranged on the right and on the left, respectively. The index *j* is the tooth number in the dental row and changes from 1 to 8. The periodontium rigidity of the *ij*th support tooth is defined as follows:

$$C_{ij} = KF_{ij}, \tag{1}$$

where $K = 1.55$ N/mm^3 is a proportionality factor [2], F_{ij} is the area of the root surface of the *ij*th tooth. The force R_{ij} arising in the *ij*th support tooth with rigidity of the periodontium C_{ij} is

$$R_{ij} = C_{ij}\delta_z, \tag{2}$$

where δ_z is the displacement of the *ij*th tooth.

The mucous tunic undergoes pressure from the prosthesis bed. The unit rigidity of the area of the mucous tunic is as follows [2]:

$$C_s = E_s / L_s, \tag{3}$$

where L_s and $E_s = 10$ MPa are the thickness and elastic modulus of the mucous tunic [5], respectively. The reaction of the mucous tunic R_s is determined with (3) as an integral of the projection of the prosthesis bed S onto the occlusal plane $x0y$ as follows:

$$R_s = \iint\limits_{(s)} \delta_z C_s dxdy. \tag{4}$$

The reaction of the mucous tunic produces moments M_x and M_y about the x and y axes, which are determined by

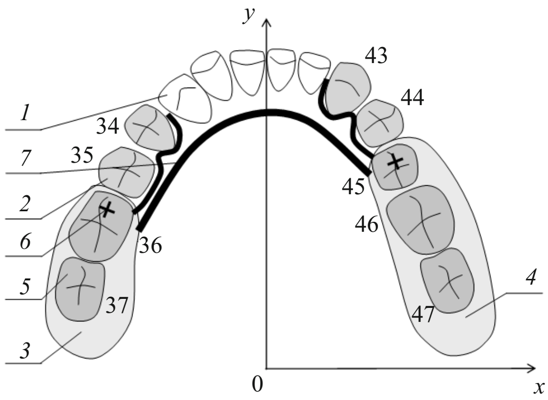


Fig. 2

$$M_x = \iint\limits_{(s)} \delta_z y C_s dxdy, \qquad M_y = \iint\limits_{(s)} \delta_z x C_s dxdy. \tag{5}$$

2. Support Reactions of a Removable Partial Prosthesis with Rigid Locks. A rigid lock consists of a patrix and a matrix attached to the support and the denture (Fig. 1). Once the patrix and matrix have been connected, the rigid support–prosthesis connection is formed, which transmits the transverse forces N_y and N_z, axial force N_x and moments M_y, M_z, and M_x.

Consider tooth replacement in the lower jaw (Fig. 2) using a prosthesis equipped with rigid locks. The intact frontal teeth of the patient are designated in Fig. 2 with number *1*. The 34 and 35th teeth on the left and the 43 and 44th teeth on the right are covered with crowns *2* connected with the patrixes of locking devices. The locking matrixes are attached to prostheses *3* of artificial teeth *4* supported on the alveolar process. The center of locking connection *6* is designated in Fig. 2 with a cross. The right and left sides of the prosthesis are connected using rigid arch *7*.

Since a human being chews by one side of the jaw, the piece of food transmitting the chewing load of the denture may be located at an arbitrary point of the dental row [8]. Assume that the denture undergoes the chewing load at a point $P(x_P, y_P)$ and the reaction of the support teeth R_{ij} applied at the coordinates of the centers of the chewing surface of the teeth (x_{ij}, y_{ij}) $(ij = 34,$ 35. 43, 44) and the reaction of the mucous tunic *S* caused by the prosthesis beds. Due to the action of the chewing load, the periodontum of the support teeth and mucous tunic below the prosthesis beds are deformed. Neglecting the small displacements of the prosthesis along the axes *x* and *y*, we suppose that it moves only vertically at a distance δ_Π and turns through an arbitrary angle about an axis lying in the occlusal plane [2].

The denture turn can be decomposed into rotation angles φ_Π and ψ_Π about the *x*- and *y*-axes. The vertical displacement of the denture point with coordinates (x, y) is described by

$$\delta_z = \delta_\Pi + x \sin \psi_\Pi + y \sin \varphi_\Pi. \tag{6}$$

To determine the reactions of the denture supports subject to the chewing load, we derive equations by which all the forces acting on the denture along the *z*-axis and the sum of moments about the *x*- and *y*-axes are equal to zero:

$$R_{34} + R_{35} + R_{43} + R_{44} + R_s = P,$$

$$R_{34} y_{34} + R_{35} y_{35} + R_{43} y_{43} + R_{44} y_{44} + M_x = P y_p,$$

$$R_{34} x_{34} + R_{35} x_{35} + R_{43} x_{43} + R_{44} x_{44} + M_y = P x_p, \tag{7}$$

where M_x and M_y are the moments produced by the mucous tunic at the denture beds about the axes $0x$ and $0y$.

In determining the reactions of the denture supports, we consider the translational displacement δ_Π and rotation angles φ_Π and ψ_Π as the main unknowns.

Expressing the loads R_s and moments M_x and M_y appearing in (7), with (4) and (5), in terms of the displacements and rotation angles, we obtain a system of three equations for δ_Π, φ_Π, and ψ_Π.

TABLE 1

Location of piece of food on tooth	Support reaction, N					
	35	34	43	44	Bed 3	Bed 4
37	2.72	0.62	–0.57	–0.05	7.67	1.6
36	2.70	2.25	–0.72	–0.75	6.12	0.40
35	2.49	2.6	0.86	0.39	3.60	0.04
34	2.27	2.71	1.8	1.09	1.97	0.15
43	0.66	1.52	3.43	2.81	0	1.57
44	0.33	1.02	3.15	2.8	0	2.70
45	0.11	0.49	2.56	2.57	0	4.28
46	–0.24	–0.26	1.69	2.19	0.38	6.25
47	–0.47	–0.98	0.42	1.43	1.91	7.68

Resolving system (7) for the displacements and substituting the results into (6), (2), and (4), we obtain the reactions of the denture supports. Table 1 collects the calculated support reactions of denture beds 3 and 4 and the support teeth. It is assumed that a vertical chewing load of 10 N is applied at the middle of the support surface of one of the 37, 36, 35, 34, 43, 44, 45, 46, and 47th teeth. The area of the surface of teeth roots F_{ij} can be found in [2], while the thickness of the mucous tunic is assumed equal to 5 mm.

Figure 3 shows the support reaction of the teeth and prosthesis beds acted upon by a chewing load of 10 N on the 36th tooth. Here the thick arrow shows the direction in which the chewing load caused by the piece of food acts. The thin arrows show the support reactions of the denture. In this case, a maximal load of 6.12 N acts on support bed 3, while the minimal one acts on support bed 4.

The supporting 44 and 43th teeth undergo a tensile force that prevents release of the prosthesis. This load is a risk factor in prosthetics because teeth, which demonstrate increased mobility and bare roots, take it badly. The vertical displacement of the point of the load application is equal to 0.01 mm, while the rotation angles φ_{Π} and ψ_{Π} are 0.7° and 0.6°, respectively. As is seen from Table 1, prosthesis bed 3 is free of load when the peace of food is on the 43, 44, and 45th teeth.

3. Support Reactions of a Removable Partial Denture with Labile Locks. Figure 4*a* schematizes a removable partial prosthesis equipped with two labile locks (Fig. 4*b*) that attach the prosthesis to support beds 3 and 4. The labile lock consists of a spheroidal patrix attached to the support teeth and a matrix attached to the prosthesis.

The lock matrixes are connected to each other and to the prosthesis with a rigid clasp. Such an attachment of the prosthesis to the support teeth makes it possible to transmit from the prosthesis to the beds the loads N_{yi}, N_{zi}, N_{ξ} $(i = 3, 4)$, whose vectors come through the sphere center, and the moments M_z and M_x (Fig. 4*c*). The centers of the spheroidal locks are located at the points (x_3, y_3) and (x_4, y_4). The prosthesis shown in Fig. 4*a* does not transmit the moment M_y acting about the *y*-axis. If the chewing load is directed to the support teeth, the mucous tunic remains unprotected.

Let us derive the equations of the static equilibrium of supports 3 and 4, that is, the forces acting along the *z*-axis and the moments about the *y*- and *x*-axes. The chewing force is applied at the point (x_{ij}, y_{ij}). Beds 4 and 3 are acted upon by the vertical forces from the prosthesis and the support reactions from the teeth, which are defined as follows:

$$R_{43} + R_{44} + N_{z4} = 0, \quad R_{43} y_{43} + R_{44} y_{44} + N_4 y_4 = 0, \quad R_{43} x_{43} + R_{44} x_{44} + N_{z4} x_4 = 0,$$

$$R_{33} + R_{34} + N_{z3} = 0, \quad R_{43} y_{43} + R_{44} y_{44} + N_{z4} y_4 = 0, \quad R_{43} x_{43} + R_{44} x_{44} + N_4 x_4 = 0. \tag{8}$$

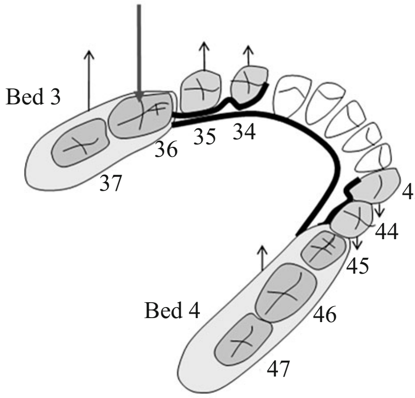
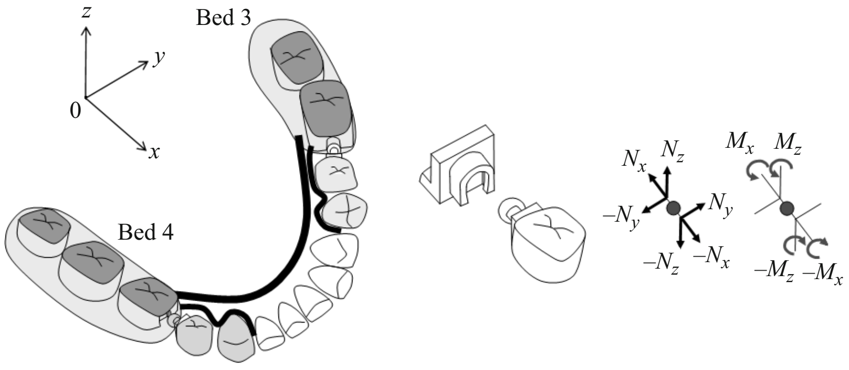


Fig. 3



a *b* *c*

Fig. 4

Under the chewing load, supports 3 and 4 perform vertical translational displacements δ_3 and δ_4 and rotate about an axis in the occlusal plane. Let us decompose the rotation of the supports into the rotation angles φ_3 and φ_4 about the *x*-axis and the rotation angles ψ_3 and ψ_4 about the *y*-axis. The vertical components of the displacements of the support points z_3 and z_4 with coordinates (x, y) can be expressed as

$$z_3 = \delta_3 + x \sin \psi_3 + y \sin \varphi_3,$$

$$z_4 = \delta_4 + x \sin \psi_4 + y \sin \varphi_4. \tag{9}$$

The force R_{ij} acting on the *ij*th support tooth with periodontium rigidity C_{ij} is determined using Eqs. (2) and (9). The prosthesis is acted upon by the chewing load *P* and by the forces $-(N_3)$ and $-(N_4)$ from supports 3 and 4. The equation of static equilibrium for the prosthesis is

$$R_s - N_{z3} - N_{z4} = P,$$

$$M_x - N_{z3} y_3 - N_{z4} y_4 = P y_P,$$

$$M_y - N_{z3} x_3 - N_{z4} x_4 = P x_P, \tag{10}$$

where M_x and M_y are the moments producing reactions of the prostheses beds about the axes 0*x* and 0*y*.

Since the nine equations (8) and (10) include 11 unknowns, the above system should be supplemented with the following conditions of equality of the displacements of the bed and prosthesis lock centers:

$$\delta_3 + x_3 \sin \psi_3 + y_3 \sin \varphi_3 = \delta_\Pi + x_3 \sin \psi_\Pi + y_3 \sin \varphi_\Pi,$$

$$\delta_4 + x_4 \sin \psi_4 + y_4 \sin \varphi_4 = \delta_\Pi + x_4 \sin \psi_\Pi + y_4 \sin \varphi_\Pi. \tag{11}$$

Substituting the expressions for displacements from (6) and rotation angles into (8) and (10), we obtain a system of 11 equations for the generalized displacements δ_Π, φ_Π, and ψ_Π. Solving this system for displacements and substituting them into (2) and (4), we get the support reactions of the prosthesis (Table 2).

From Table 2, we can see that if the food piece is located on the 35, 34, 43, and 44th teeth, they take completely the chewing load, while the other teeth and prosthesis beds are free of loads. When the food piece presses on the 36, 37, 45, 46, and 47th teeth, the support reactions are distributed between the prosthesis beds and the support teeth. The distributions of the support reactions of the removable partial denture with rigid and labile locks in the case where the food piece is on the 46th tooth are compared in Fig. 5. As we can see, with labile locks, the support reactions are distributed more nonuniformly. Thus, the overloading of the support beds and support teeth is more probable with labile locks than with rigid locks.

Conclusions. The compensation of double-sided distally unlimited defects of dental rows is one of the most complicated problems of orthopedic stomatology. Such defects are compensated using removable partial dentures with rigid or labile locking devices, which fix the denture in the dental row. The length of defects and the presence of teeth with increased

TABLE 2

Location of piece of food on denture	Supporting reaction, N					
	35	34	43	44	Bed 3	Bed 4
37	1.63	–0.77	–0.27	0.12	8.97	0.31
36	2.31	–1.09	1.15	–0.5	6.57	1.56
35	10	0	0	0	0	0
34	0	10	0	0	0	0
43	0	0	10	0	0	0
44	0	0	0	10	0	0
45	1.31	–0.62	2.69	–1.17	1.3	6.49
46	0.36	–0.17	1.79	–0.78	0.38	8.41
47	–0.87	0.41	0.34	–0.15	1.02	9.26

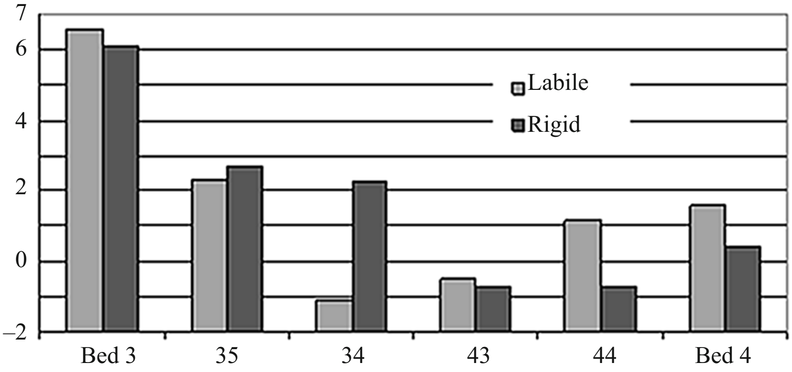


Fig. 5

mobility and bare roots near them are risk factors for the prosthetic treatment with removable partial prostheses. To prevent overloading of a dental apparatus and enable safe fixation of the prosthesis, it is necessary to estimate the possible support reactions at the design stage.

We have determined the support reactions of prostheses with rigid and labile locking devices. It has been shown that when chewing, the support teeth undergo the action of tensile forces poorly taken by the teeth due to their increased mobility, the presence of bare roots, and considerable dependence of the support reactions on the type of the locking system. In using removable partial prostheses with rigid locks, the support reactions acting on the teeth and alveolar process are distributed more uniformly than in the case of the same prostheses with labile locks. For this reason, overloading of the support beds and support teeth with labile locks is more probable than with rigid locks.

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